A Design Margin Index for MTBF and Ao

Joseph A. Dzekevich, Raytheon

Jeffrey Garretson, Raytheon

Key Words: CGI, Cp, Cpk, CPM, Design Margin Index, Design Margin, DFR, DFSS

SUMMARY & CONCLUSIONS

One of the hallmarks of a Design For Six Sigma process is Critical Parameter Management. The Critical Parameter Management methodology uses various techniques to assess and manage critical design parameters. One such measure is a Capability Growth Index. The purpose of Capability Growth Index is to give designers and development managers an early heads-up as to where they stand in achieving the metric under scrutiny, such as MTBF. The Capability Growth Index measure is based on the widely used quality Process Capability indices, Cp and Cpk. The problem reliability practitioners have in the area of Design For Reliability is that the major reliability metrics are Mean Time Between Failure, Mean Time To Repair and Operational Availability. Mean Time Between Failure, Mean Time To Repair and Operational Availability are moments of a distribution, are usually not s-normal, and thus, a Capability Growth Index based upon a Process Capability methodology will lead to incorrect results.

This paper examines the development of a Design Margin Index for use with Mean Time Between Failure and Operational Availability.

1 BACKGROUND

The Critical Parameter Management, CPM, methodology uses various techniques to assess and manage critical design parameters, including Capability Growth Indices. As introduced by Creveling, Slutsky and Antis [1] "the capability growth index quantifies the aggregate functional capability maturity of a given subsystem...the capability is measured in units of percent. The percentage is the aggregated amount of subsystem Critical Function Responses that are at their goal of Cp =2."

In other words, Capability Growth Index, CGI, was invented to give designers and development managers an early advisory as to where they stand in achieving the metric under scrutiny such as MTBF. The CGI metric is based on using the widely used quality metric Cp or Cpk (Process Capability indices). The problem we have as reliability practitioners in the area of Design For Reliability is that our main metrics are things like Mean Time Between Failure, Mean Time To Repair and Operational Availability. The Cp and Cpk indices are based on individually measured item samples, such as weight, height, width, density and speed. However, Key Performance Parameters such as MTBF and MTTR are distribution metrics and are not individual items to be measured, using a CGI based index on Cp or Cpk is not valid, because one should not apply Cp and Cpk to distribution means that already carry variation within their values. Then how does the reliability practitioner solve this problem?

2 THE RELIABILITY DESIGN CHALLENGE

How do we know when a design is successful, from a Reliability, Maintainability, and Availability point of view?

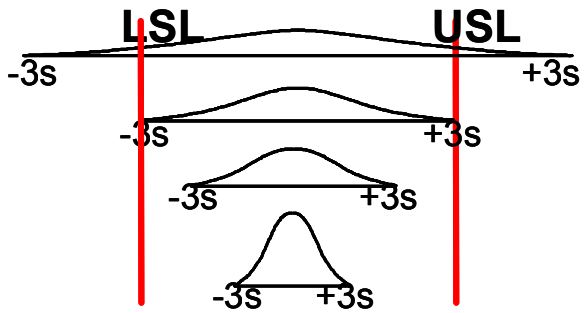
1. It just meets specification.
2. It is robust to variation and unknown margins.
3. It has sufficient design margins against a margin goal, which is a performance metric.

Thus, the design challenge is: how do we measure the maturity of MTBF, MTTR and Ao metrics during the course of a development project?

3 PROCESS CAPABILITY INDICES

Figure 1 Process Capability and Specification Limits

Cp and Cpk are indices that measure the statistical



capability of a process. Figure 1 shows various Cp possibilities, where the worst process capability is at the top and the best process capability is at the bottom. The top process will have defective parts because there are process tails outside of the specification limits. The second process just meets specification without design margin (no margin). The third process has some and the fourth process has a lot of design margin.

In Figure 2, processes A and B each have a Cp equal to 1.5, yet process B will have defectives because it is not centered at the nominal specification. Cp is calculated as the specification width divided by the process spread; 6 sigma processes must have a Cp of 6 or better.

Cpk was developed to take into account the fact that a process centerline can shift. Cpk looks to see how close the

ends of the distribution tails are to the nearer of the high or low specification limits.

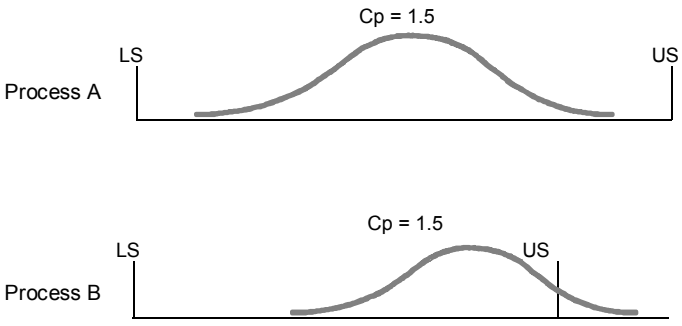


Figure 2 Limitation of Cp Index

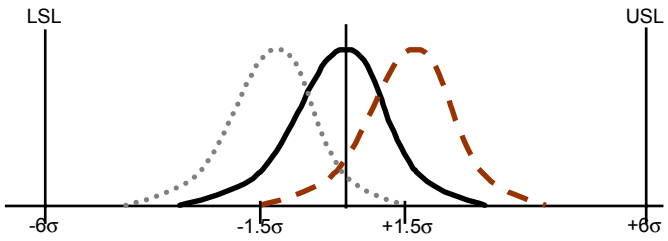


Figure 3 The Effects of Using Cpk

Figure 3 shows that Cpk takes into account process shifting.

4 CFR, CGI AND DMI

The first step is to set a design margin goal. Cp and Cpk are then used to calculate Design Margin Indices, DMI [1]. The purpose of Critical Parameter Management is to manage Critical Function Responses. We want to know at various product development phases if the design requirement of interest has margin per the set goal, and if not, how much we must we improve?

The equation for Cpk takes the form:

$$Cpk = \min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] \quad (1)$$

The equation for targeted DMI (the goal) is:

$$DMIt = \frac{\min(USL-TN,\ TN-LSL)}{3\times TMV} \quad (2)$$

DMIt = Design Margin Index Target (goal)
TN = Target Nominal
TMV = Target Maximum Variability
The equation for the predicted DMI takes the form:

$$DMIp = \frac{MIN(USL-\mu p, \mu p - LSL)}{3\times \sigma p} \quad (3)$$

From equations 1, 2, and 3, DMI is based upon Cpk. and thus, the s-normal distribution. For example:

- If a product's target Cpk margin is 1.33.
- And the product's nominal specification power is 28.18 watts.
- And the current prediction is 30.5 watts with a standard deviation of 0.6 watts.

DMIt = 1.33
DMIp = (30.5 - 28.18)/(3*0.6) = 1.29
DMI = DMIp / DMIt = 1.29 / 1.33 = 0.97

The DMI indicates that our design has utilized 97% of our Design Margin target. DMI will equal 1 if we meet our DMIt goal of 1.33. The good news is that margins are being managed and variability is understood. The downside is that this calculation assumes an s-normal distribution.

But what happens to Cpk if the distribution is non-normal, (skewed) as in Figure 4?

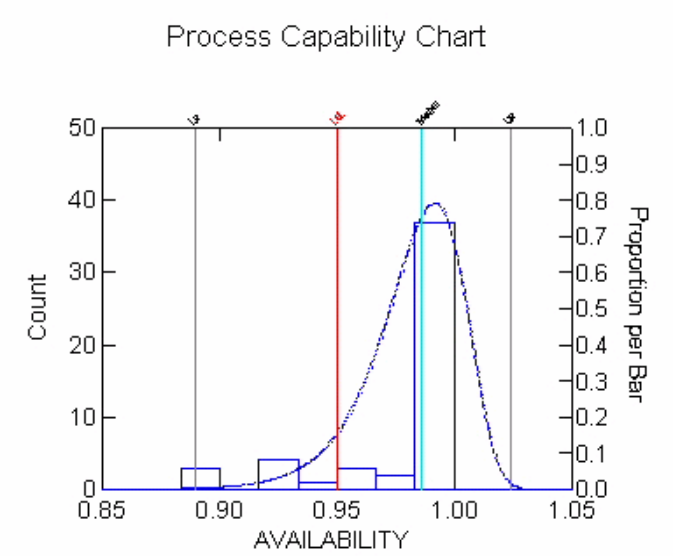


Figure 4 Distribution Shape Affects Cpk

For the distribution [2] shown in Figure 4:

- The calculated Cpk for the s-normal distribution is .31.
- The calculated Cpk for the Weibull distribution is .37.
- The calculated Cpk for a Pearson is .404.
- The calculated Cpk for a skewed distribution is: .408

The Cpk changes, as expected, when taking into account that the distribution is not s-normal.

5 CALCULATING DMI FOR AO

There are two fundamental problems with trying to use Cpk based Design Margin Index methodology on Operational Availability and Mean Time Between Failure.

1. Ao and MTBF are moments of a distribution. They are not individual measured points.
2. The Ao and MTBF distributions are not s-normal.

For example: A Monte Carlo simulation of operational availability [3] was conducted on a system. The simulation provided an average Ao estimate of 0.9812 with a standard deviation of 0.032113. If the required Ao is specified at 0.95, do we have enough margins for a 6 sigma design?

The output of the simulation is shown in the Probability Density Function in Figure 4. The best fit was a Weibull

distribution. If we calculate DMI using the Cpk methodology, it would look like this:

- CpK = (.9812 - .95) / (3*.032113) = .3238
- DMI = .3218 / 2 = .16

A DMI of 0.16 is not very good because we want a DMI of 1.0 which says that we met our margin goal. A DMI of 0.16 says that we have very little margin, if we wish this to be a 6 sigma design. But, we have to remember that the distribution is not s-normal and Cpk is intended for measuring individual items and not a collection of Ao means from a distribution.

The recommended alternative calculation method is to treat the Monte Carlo Ao dataset as a sampling distribution and then apply the t-distribution or to curve fit the data to a distribution type and calculate a fraction nonconforming in order to obtain a more realistic DMI metric.

Using Microsoft Excel nomenclature, the recommended process is this:

- Treat the 50 runs as a sampling distribution
- Calculate t critical:
  - = (Ao Mean-Ao Req) * SQRT(No. Runs) / SD Runs
  - =(.9812 - .95)*SQRT(50)/0.032113 = 6.87003132
- Calculate probability of failure from t-value
  - =TDIST(t critical,(No. Runs-1),1)
  - =TDIST(6.87003132,(50-1),1) = 5.2733E-09
- Calculate the 6 sigma value, using above as Defects per Unit
  - =-NORMSINV(Prob Failure/2)
  - =-NORMSINV(5.2733E-09/2) = 5.83830986
- Calculate DMI:
  - DMI = 5.83830986/6 = 0.97305

Taking into account that the data is a distribution, and not a set of individual measured items and that the distribution is not s-normal, the new DMI is 0.97 against a goal of 1.0 instead of the 0.16 that was obtained using the Cpk based methodology. This means that if we were to accept the 0.16, we would probably greatly over-design the system trying to get it up to the 1.0 goal.

6 CALCULATING DMI FOR MTBF

The procedure for calculating a DMI for a MTBF is very similar to the method we used to calculate a DMI for Ao.

To illustrate, let us assume the following:

- A MTBF goal of 40,000 hours.
- A product prediction of 50,000 hours.

The recommended steps for the analysis are:

- Run a statistical bootstrap or simulation to generate a 50,000 hour dataset.
- Calculate t critical = (MEAN - SPEC) * SQRT(sample size)/SD
- Find the probability of failure from T = TDIST(T critical, (sample size - 1), 1)
- Standardized the Six Sigma Value = -1 * NORMSINV(Prob of Failure from T / 2)
- Calculate DMI = Standardized Six Sigma Value / 6

Tables 1 and 2 summarize the results of the statistical bootstrapping procedure and the DMI calculations.

Table 1 shows that the bootstrapping procedure [2] did not give us exactly 50,000 hours, so as we proceed with this example, we will use the 49,351 hour results as our product MTBF.

Exponential Sample Mean	49,351.50
Exponential Sample Standard Deviation	4,734.80
90% UCL 2x	50,590.26
90% LCL 2x	48,112.74
90% LCL 1x	48,494.41
3 Sigma UCL	51,360.31
3 Sigma LCL	47,342.69

Table 1 MTBF Bootstrap Results for 50,000 hour MTBF

Table 2 shows the results of calculating the DMI for the 49,351 hour MTBF.

If specification was:	40,000
Calculated t Value (t critical):	13.96576169
Probability of Failure from t Value:	5.21255E-19
Six Sigma value, using above as Defects per Unit:	8.907650424
DMI from Sigma Value:	**1.484608404**

Table 2 MTBF DMI Calculation Results

The conclusion we draw from Table 2 is that our design, with a MTBF of 49,351 hours, not only exceeds the requirement, it exceeds 6 sigma, because the DMI is greater than 1.0. If the product is over designed for its application, cost reduction opportunities may be possible.

7 SUMMARY AND CONCLUSIONS

For good design margin goals, it has been shown that we want a Design Margin Index, of 1.0.

It was also shown that Design Margin Indices based on Cp and/or Cpk can only be used on individual item measurements and that they should not be used on parameters such as MTBF, MTTR and Ao.. Mean Time Between Failure, Mean Time To Repair and Operational Availability are moments of distributions and are usually not s-normal, and as distribution parameters, they include variation, which defeats the purpose of using Cpk.

One can use Monte Carlo [3] and/or bootstrapping [2] techniques to generate Ao and MTBF datasets to create distributions to be used for the Design Margin Index metric calculation. These newly generated datasets are then treated as sampling distributions or one can curve fit the data. The t-distribution or a fitted distribution can then be applied to find the fraction nonconforming, which then can be used to calculate Design Margin Index. Otherwise, one may end-up grossly over or under designing the product's reliability margins.

Postscript: It should be also noted that a Cp or Cpk based DMI can not be used for metrics that do not have linear measurements, such as dB margin in a communications link.

8 ACRONYMS

CFR	Critical Function Responses
CGI	Capability Growth Index

Cp	Process Capability
Cpk	Process Capability, relative to the closest specification limit
CPM	Critical Parameter Management
DMI	Design Margin Index
DMIp	Design Margin Index xxx
DMIt	Design Margin Index Target
DFR	Design For Reliability
DFSS	Design For Six Sigma
MTBF	Mean Time Between Failure
MTTR	Mean Time To Repair
S-Normal	Statistically Normally Distributed
TPM	Technical Performance Metrics

REFERENCES

1. C. M Creveling, J. L. Slutsky, D. Antis, Jr. "Design for Six Sigma in Technology and Product Development," Prentice Hall PTR, 2003, ISBN 0-13-009223-1
2. The Figure 4 distribution and Table 1 Bootstrapping results were produced using the SYSTAT software package. http://www.systat.com
3. Monte Carlo simulations were developed and run using the SPAR modeling and simulation tool by Clockwork Solutions. http://www.clockwork-solutions.com

BIOGRAPHIES

Joseph A. Dzekevich
Raytheon
Mail Stop 3.2.3879
1001 Boston Post Road
Marlborough, MA 01752, USA

e-mail: jdzekevich@ieee.org

Joe Dzekevich is a Senior Principal Systems Reliability Engineer with Raytheon's Network Centric Business in Marlborough, MA. He has a B.S.I.T. from Northeastern University and a M.B.A. from Clark University. He is an ASQ Certified Reliability Engineer and a Senior Member of both the IEEE Reliability and the ASQ Reliability and Statistics divisions. He is also the Vice Chair of the IEEE Boston Reliability Chapter. Joe has over thirty years of experience in all aspects of reliability engineering, specializing in systems analysis, accelerated testing and applied statistical analysis.

Jeffrey Garretson
Raytheon
1001 Boston Post Road
Marlborough, MA 01752, USA

e-mail: jeg4@lehigh.edu

Jeffrey Garretson is currently a graduate student at Cornell University. He graduated from Lehigh University with a B.S. in Computer Science and a minor in Pure Mathematics in May 2006, and will earn his Master's Degree in Computer Science in May 2007. He has been able to utilize his mathematical background, while gaining experience using statistical analysis, during his internships at Raytheon during the summers of 2004, 2005, and 2006.